Λ_ϕ are more easily predictable than λ_x and λ_y over long periods of time. This is because x and y do not return to the same respective values with any regularity while the values of r and ϕ are approximately periodic. The multipliers corresponding to the movement of Jupiter out of the ecliptic plane in both sets of coordinates (the z and θ multipliers) do not exhibit a periodic behavior over the twelve and one-half month interval.

Thus, the choice of the coordinate system in which the problem is set is quite important when searching for recurrent behavior of the type presented here. It is possible that there is a coordinate system which would exhibit a periodic behavior for all six Lagrange multipliers, but the spherical polar coordinates appear adequate for guessing the initial values of the Lagrange multipliers for launch dates outside the range of values presented.

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Nonaffine Similarity Laws Inherent in Newtonian Impact Theory

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Nomenclature

C_D, C_L = drag and lift coefficients, respectively
d = body width
k = Newtonian constant
l = body length
$\bar{s}$ = upper limit on s
x,y,s = longitudinal distance, lateral distance, arc length
y' = local slope, dy/dx
θ = angle between local tangent and freestream velocity vector
λ = reference length
ξ = general complementary parameter
$\bar{\xi}$ = upper limit on ξ

Subscripts

0 = basic configuration
1 = complementary configuration
2 = doubly-complementary configuration

NEWTONIAN impact theory provides a basis for studying nonaffine similarity laws; as opposed to affine or linear similarity laws.[1–5] This Note presents those nonaffine similarity laws and transformations for two-dimensional bodies, subject to the limitations of Newtonian impact theory. Considerations are given only to flat-topped bodies at zero angle of attack, since the upper or lower portion of any nonflat-topped configuration can be treated in a similar manner. The similarity laws for complementary configurations are derived from the following Newtonian drag and lift equations

$$C_D = \frac{k}{\lambda}\int_0^d \sin^2\theta dy \tag{1}$$

$$C_L = \frac{k}{\lambda}\int_0^l \sin^2\theta dx \tag{2}$$

where θ (see Fig. 1) is the angle between the local tangent and the freestream velocity vector (applicable only for $\theta \geq 0$), k is a constant factor ($k = 2$ for the classical Newtonian equation), and λ = reference length.

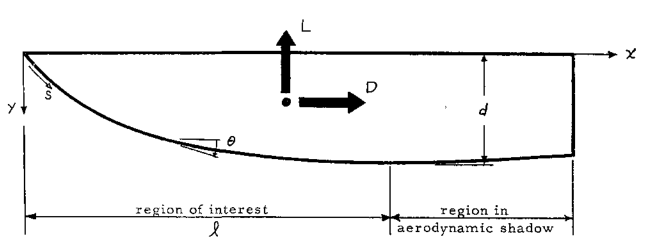

Fig. 1 Flat-topped body.

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Definitions of Complementary Configurations

Definition 1: Two configurations are said to be complementary with respect to any parameter ξ if the slopes at corresponding values of ξ have complementary angles; i.e.

$$\theta_0(\xi) + \theta_1(\xi) = \pi/2 \tag{3}$$

Note that this implies that if there is a one-to-one correspondence of θ_0 with ξ, then there is a one-to-one correspondence of θ_1 with ξ. Since $y' \equiv dy/dx = \tan\theta$, it follows from Eq. (3) that

$$y_0'(\xi)\cdot y_1'(\xi) = 1 \tag{4}$$

Definition 2: Transformations effected according to Eqs. (3) or (4) are said to be complementary transformations.

In Fig. 1, only the interval $0 \leq x \leq l$ is of concern. Therefore, only complementary configurations in this interval need be found for the Newtonian application. In other words, two configurations with complementary forebodies and unrelated afterbodies (with the restriction that $\theta < 0$ on the afterbody portion) can have the same similarity properties. These configurations are not completely complementary which gives rise to the need for the following definition.

Definition 3: Two configurations are said to be completely complementary with respect to ξ if both configurations satisfy Eq. (3) and have the same limits in ξ. If the limits overlap, then they are said to be partially complementary.

Another important definition which is used to relate many configurations is as follows.

Definition 4: If a configuration (1) is complementary to a configuration (0) with respect to ξ_1, and if a configuration (2) is complementary to configuration (1) with respect to ξ_2, then it is said that configuration (2) is doubly-complementary to configuration (0) with respect to $\xi_2\xi_1$.

From the previous definitions, it follows that the set of equations

$$y_0'(\xi_1)\cdot y_1'(\xi_1) = 1 \tag{5}$$

$$y_1'(\xi_2)\cdot y_2'(\xi_2) = 1 \tag{6}$$

defines the configurations (0) and (2) as being doubly-complementary with respect to $\xi_2\xi_1$. The extension of this definition to multiply-complementary configurations is obvious. Finally, one last definition related to multiply-complementary configurations is needed.

Definition 5: Two configurations which are related through an even (odd) number of complementary transformations are said to be even (odd)-multiple configurations.

With these definitions, the similarity laws may now be derived. Although the derivations to follow are for the parameters $\xi = x,y$, and s, other ξ parameters may also be used.

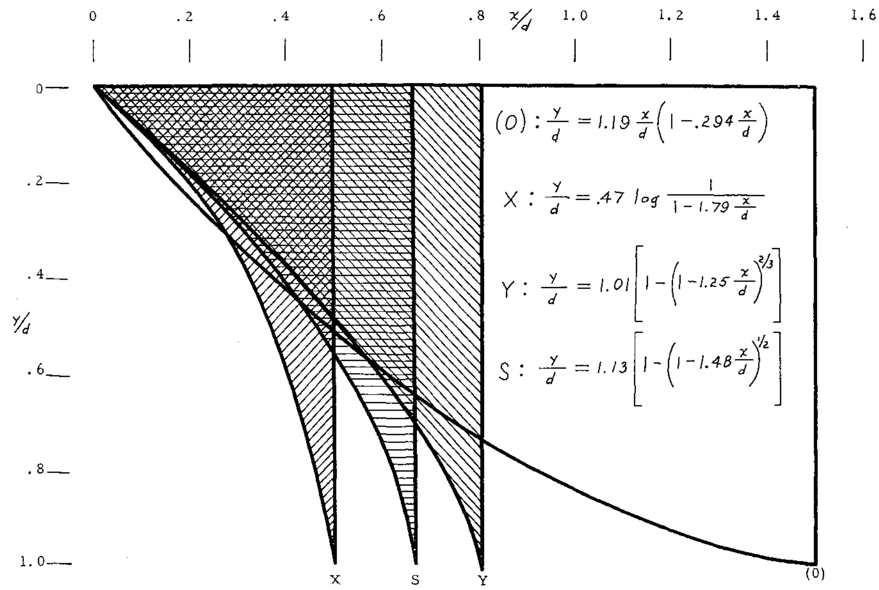

Fig. 2 Configurations complementary to a flat-topped parabolic body.

Similarity Laws

For any configuration defined as a function of ξ, Eqs. (1) and (2) may be rewritten as follows:

$$\frac{\lambda}{k}C_D = \int_0^{\bar{\xi}} \sin^2\theta \frac{dy}{d\xi} d\xi \tag{7}$$

$$\frac{\lambda}{k}C_L = \int_0^{\bar{\xi}} \sin^2\theta \frac{dx}{d\xi} d\xi \tag{8}$$

These equations will now be applied to specific ξ parameters.

A. Configurations complementary with respect to x

From Eq. (7), the general drag expression for $\xi = x$ and the upper limit $\bar{\xi} = l$ is

$$\frac{\lambda}{k}C_D = \int_0^l \sin^2\theta y' dx = y(l) - \int_0^l \sin\theta \cos\theta dx \tag{9}$$

For any two complementary configurations, Eqs. (3) and (9) reduce to

$$(\lambda_0/k_0)C_{D_0} - y_0(l) = (\lambda_1/k_1)C_{D_1} - y_1(l) \tag{10}$$

This relation is the similarity law which relates the drag on two complementary configurations defined by Eq. (4).

For the lift, Eqs. (3) and (8) yield

$$(\lambda_0/k_0)C_{L_0} + (\lambda_1/k_1)C_{L_1} = l \tag{11}$$

B. Configurations complementary with respect to y

From Eqs. (3) and (7), the similarity law for $\xi = y$ and $\bar{\xi} = d$ is

$$(\lambda_0/k_0)C_{D_0} + (\lambda_1/k_1)C_{D_1} = d \tag{12}$$

For the lift, Eqs. (3) and (8) with $\xi = y$ become

$$(\lambda_0/k_0)C_{L_0} = (\lambda_1/k_1)C_{L_1} \tag{13}$$

C. Configurations complementary with respect to s

From Eqs. (3, 7, and 8), the similarity law for $\xi = s$ and $\bar{\xi} = \bar{s}$ is

$$(\lambda_0/k_0)C_{D_0} + (\lambda_1/k_1)C_{L_1} = y_0(\bar{s}) = x_1(\bar{s}) \tag{14}$$

The interesting result about this similarity law is that the drag and lift are related to one another.

In the same manner, the following similarity law is also derived

$$(\lambda_1/k_1)C_{D_1} + (\lambda_0/k_0)C_{L_0} = y_1(\bar{s}) = x_0(\bar{s}) \tag{15}$$

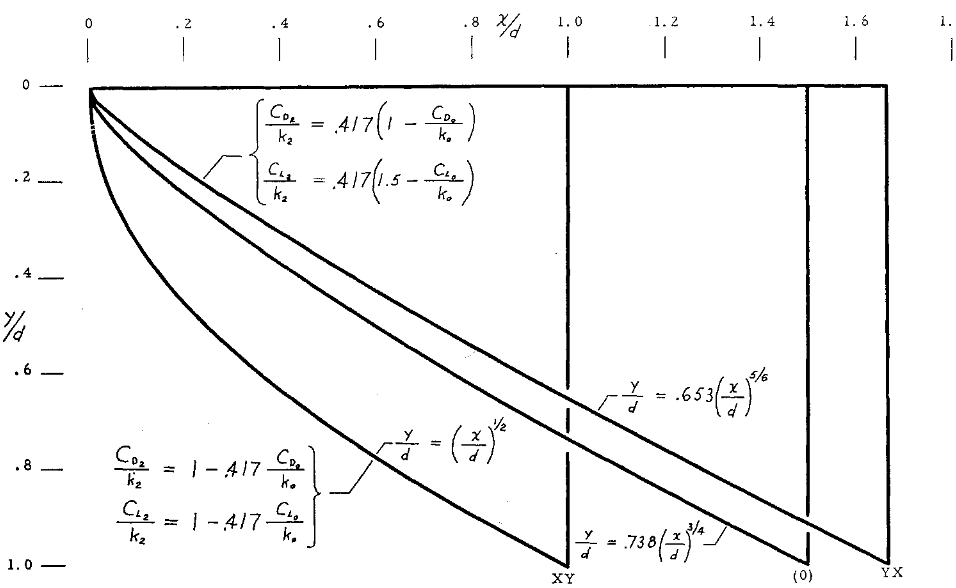

Fig. 3 Configurations doubly-complementary to a flat-topped power-law body.

D. Summary of similarity laws

The symmetry of the nonaffine similarity laws derived above becomes apparent when comparing Eqs. (10) and (13), Eqs. (11) and (12), Eqs. (14) and (15). Eqs. (10) and (13) state that the force coefficient (with the appropriate reference length and Newtonian constants) in the direction of the complementary parameter ξ less the maximum width (distance normal to the free-stream vector) is invariant. Eqs. (11) and (12) state that the sum of the force coefficients normal to the complementary parameter ξ (with the appropriate reference lengths and Newtonian constants) is equal to the maximum value of this parameter, $\bar{\xi}$. The symmetry of Eqs. (14) and (15) is obvious.

Complementary Configurations

To find complementary configurations for which the similarity laws of the previous section apply, Eq. (4) may be rewritten as follows:

$$y_1'(\xi) = 1/y_0'(\xi)$$

Integrating this equation for configurations complementary with respect to x, there results

$$y_1(x) = \int_0^x \frac{dx}{y_0'(x)} \tag{16}$$

and for configurations complementary with respect to y

$$x_1(y) = \int_0^y y_0'(y)dy \tag{17}$$

For configurations complementary with respect to s, it can be shown that the following transformation is effected

$$(x_0,y_0) \rightarrow (y_1,x_1) \tag{18}$$

Examples of complementary configurations obtained by applying Eqs. (16–18) to a flat-topped parabolic body with a leading-edge angle of 50° and a fineness ratio of 1.5 are shown in Fig. 2. Note that the transformed bodies are concave; whereas the original is convex. This is inherent in single transformations of complementary configurations.

Multiply-complementary configurations arise when more than one transformation is effected. For even-multiple configurations, the transformations are convex-to-convex and concave-to-concave; and for odd-multiple configurations, the transformations are convex-to-concave and concave-to-convex. (Since Newtonian theory may have limited application to concave bodies, the even multiplies for convex bodies are of practical interest.) Note that even-multiple configurations have the same leading-edge angle. Therefore, their Newtonian constants may be taken to be equal; i.e., $k_0 = k_2 = k_4 \ldots$

Examples of multiply-complementary configurations are shown in Fig. 3 for a power-law body with exponent $\frac{3}{4}$. Note that the complementary configurations are also power-law bodies. In general (with some exceptions), power-law bodies remain power-law bodies under complementary transformations.

Conclusions

Nonaffine similarity laws and transformations have been derived, subject to the limitations of Newtonian impact theory. Applying these transformations to different configurations, the lift and drag coefficients of complementary configurations can then be related through the appropriate similarity laws. The results of this paper may be extended to three-dimensional configurations and generalized to configurations which are not complementary (i.e., $\theta_0 + \theta_1 \neq \pi/2$).

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A Molecular Model for Tangential Momentum Accommodation

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The effectiveness of tangential momentum transfer from a flowing gas to a boundary surface is generally described in terms of a tangential momentum accommodation coefficient σ defined as the fraction of the flux of tangential momentum transmitted to the surface by molecular collisions or, equivalently, the fraction of molecules that are diffusely reflected from the surface. Values of σ for a number of gas-solid systems have been obtained from measurements of slip velocity and surface drag.[1–4] The purpose of this Note is to report calculated values of σ for an idealized, monocrystalline model of the gas-solid interface and to compare these with the experimental values. From this comparison the effect of the polycrystallinity of real surfaces on tangential momentum transport can be inferred. It is also shown that the values of σ obtained from the slip velocity for the model system are different, in general, from those determined from the calculated surface drag. This suggests that experimental values of the accommodation coefficient for certain real surfaces may also depend on which of the two quantities is measured. The results of the study are most applicable to the aerodynamically interesting case of high-speed, low-density gas flows, although at least qualitative inferences can be drawn for other flow systems.

Consider a rarefied gas confined to the space $y > 0$ by an infinite flat plate at $y = 0$. The plate is stationary and the gas flows parallel to it with a velocity $\mathbf{u} = \hat{\mathbf{x}}u(y,t)$. We assume that far from the plate the flow is steady and uniform; thus

$$\lim_{y\to\infty} u(y,t) = U$$

a constant. The molecular state of the gas is given by $f(\mathbf{c},y,t)$, the local distribution function for molecular velocities $\mathbf{c}$, and the macroscopic properties of the gas are moments of f. In particular, the gas velocity at the surface and the interfacial momentum flux are given by

$$\mathbf{u}_s = n_s^{-1}\int \mathbf{c} f_s d\mathbf{c} \tag{1a}$$

and

$$\boldsymbol{p}_s = \int m(\mathbf{c} - \mathbf{u}_s)(\mathbf{c} - \mathbf{u}_s) f_s d\mathbf{c} \tag{1b}$$

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